Higgs amplitude mode in the vicinity of a quantum critical point

Nicolas Dupuis

Laboratoire de Physique Théorique de la Matière Condensée
Sorbonne Université & CNRS, Paris

Coll.: A. Rançon (Université de Lille, France)
F. Rose (TUM, Munich, Germany)

A. Rançon and ND, PRB 2014
F. Rose, F. Leonard and ND, PRB 2015
F. Rose and ND, arXiv:1801.03118

Nature Physics **11**, 93 (2015)

news & views

SUPERCONDUCTIVITY

Higgs, Anderson and all that

The Higgs mechanism is normally associated with high energy physics, but its roots lie in superconductivity. And now there is evidence for a Higgs mode in disordered superconductors near the superconductor–insulator transition.

Philip W. Anderson

Anderson-Higgs mechanism in superconductors: the photon (*gauge field*) and the pairing phase field (*Goldstone boson*) combine to make a massive plasmon (*W and Z bosons*). The Higgs particle corresponds to the pairing amplitude mode.

Amplitude modes in condensed matter: Higgs modes

OUTLINE

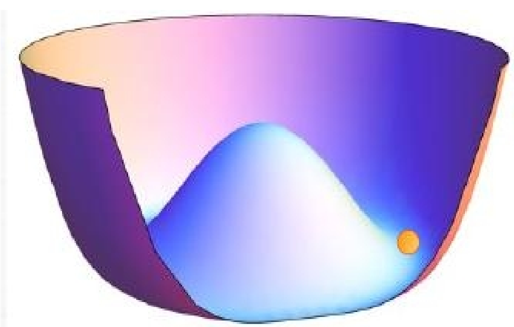
- Quantum phase transition: elementary excitations from mean-field theory
- Beyond mean-field theory
  - dimensionality
  - longitudinal/transverse vs amplitude/direction fluctuations
- Results from nonperturbative RG and QMC

Quantum phase transition

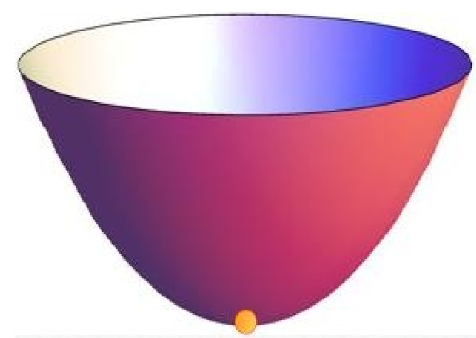
N-component order parameter: $\boldsymbol{\varphi} = (\varphi_1, \cdots, \varphi_N)$

Potential with O(*N*) symmetry: $V(\boldsymbol{\varphi}) = \delta\,\boldsymbol{\varphi}^2 + g(\boldsymbol{\varphi}^2)^2$

$\delta < 0$



$\delta > 0$



[picture: D. Podolsky APS talk 2013]

$\langle \varphi \rangle \neq 0$ — $\delta_c = 0$ — $\langle \varphi \rangle = 0$ — δ

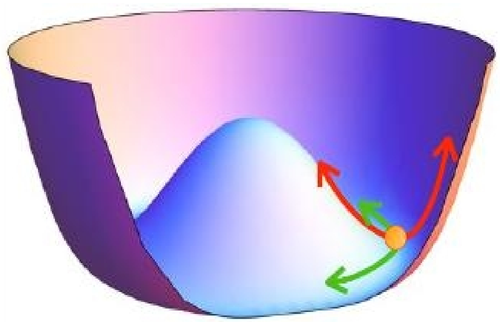
QCP

ordered phase
(spontaneous symmetry breaking)

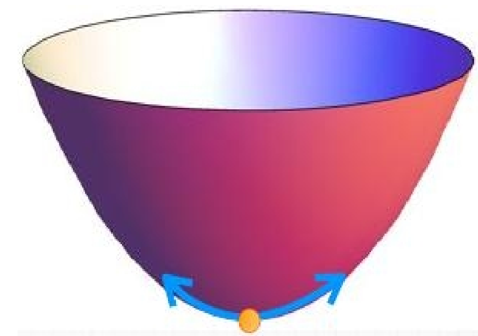
disordered phase

Dynamics (mean-field theory)

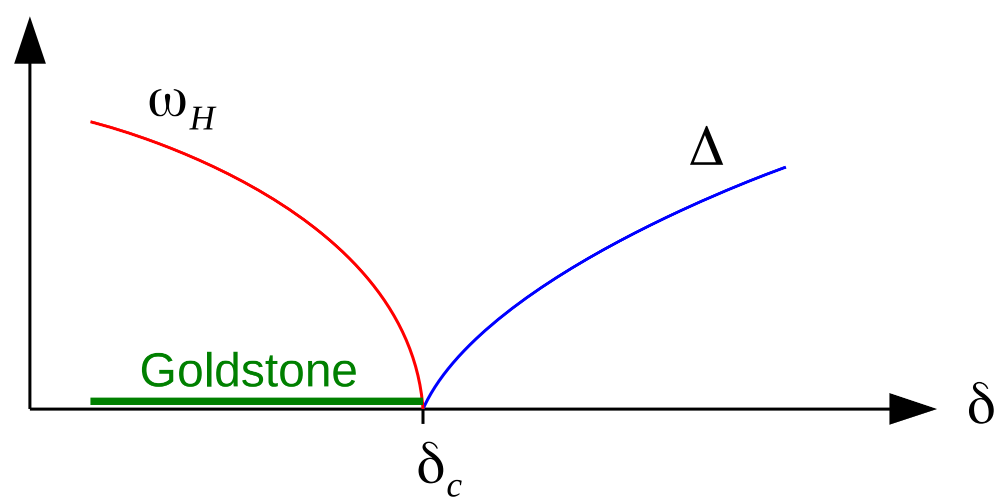
- Relativistic dynamics $$S = \int d^d r \int dt \; (\partial_t \boldsymbol{\varphi})^2 - (\nabla \boldsymbol{\varphi})^2 - V(\boldsymbol{\varphi})$$



N-1 Goldstone modes
1 amplitude (Higgs) mode



N gapped modes



$$\Delta = A|\delta - \delta_c|^{1/2}$$

$$\omega_H = \sqrt{2} A|\delta - \delta_c|^{1/2}$$

$$\frac{\omega_H}{\Delta} = \sqrt{2}$$

- Galilean-invariant bosons: Gross-Pitaevskii equation

$$\left(-i\partial_t - \mu - \frac{\nabla^2}{2m}\right)\psi + g|\psi|^2\psi = 0$$

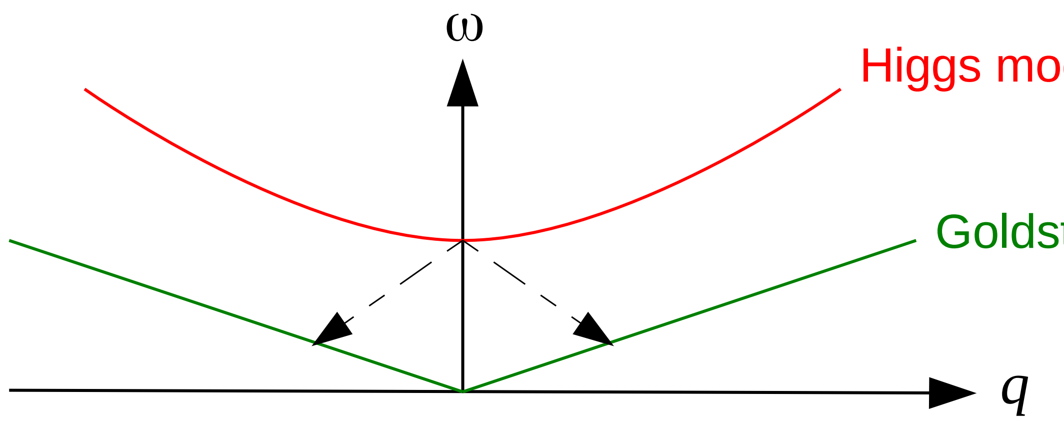
Bogoliubov sound mode: $\omega = c|\boldsymbol{q}|$ for $\boldsymbol{q} \to 0$

No Higgs mode! $[\psi, \psi^+] = 1$

- Higgs mode in (non-relativistic) condensed matter requires emergent Lorentz invariance.

Beyond mean-field

The Higgs mode can decay into Goldstone modes



- Is the Higgs mode a long-lived excitation (in particular near the QCP)?
- How can it be observed (which correlation function)?

Quantum O(N) model

$$S = \int_0^\beta d\tau \int d^d r \; \frac{1}{2}(\partial_\tau \boldsymbol{\varphi})^2 + \frac{1}{2}(\nabla \boldsymbol{\varphi})^2 + \delta \boldsymbol{\varphi}^2 + g(\boldsymbol{\varphi}^2)^2 \qquad (\beta = 1/T)$$

- generalization of classical O(N) model
- T=0: Lorentz symmetry: classical O(N) model in d+1 dimensions (dynamical critical exponent z=1, upper critical dimension d=3)
- describes critical regime of many systems
  - quantum antiferromagnets
  - Josephson junction arrays
  - granular superconductors
  - bosons in optical lattices

- *d*=3: interactions are irrelevant at QCP (Gaussian fixed point)

  Mean-field theory becomes exact as $\delta \rightarrow \delta_c$

  The Higgs mode is a long-lived excitation near the QCP

- *d*=2: interactions are relevant

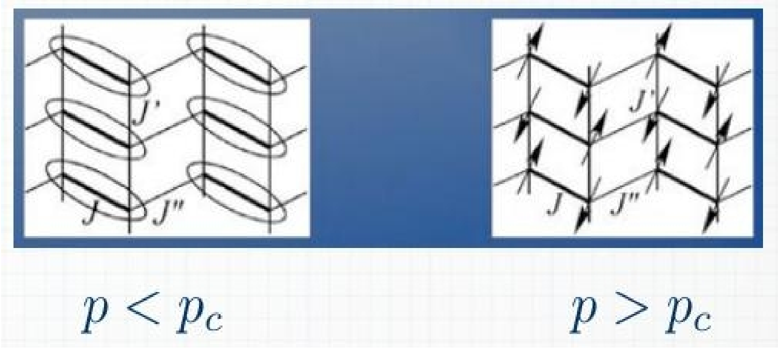
  QCP: 3D Wilson-Fisher fixed point $(\nu, \ \eta, \ z=1)$

$$T = 0: \ \text{length scale: } \xi \sim |\delta - \delta|^{-\nu}$$

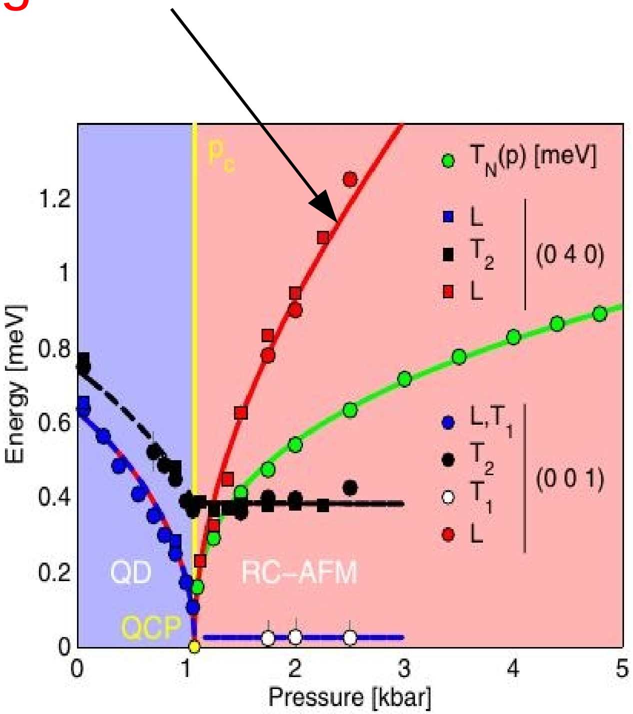
$$\text{energy scale: } \Delta \sim |\delta - \delta_c|^{z\nu}$$

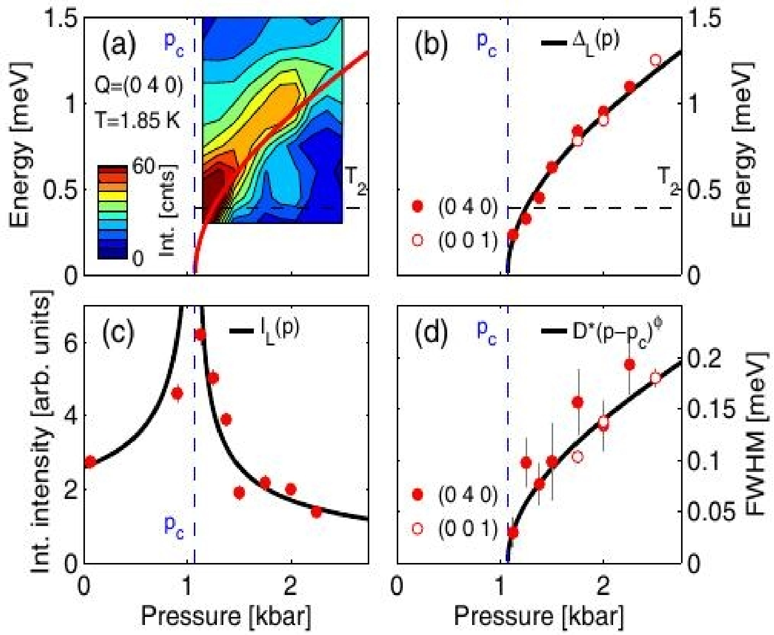
Neutron scattering in TlCuCl3 (3D)

3D spin dimer antiferromagnet
pressure-induced transition



Higgs mode

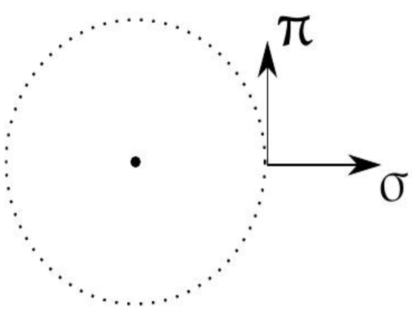




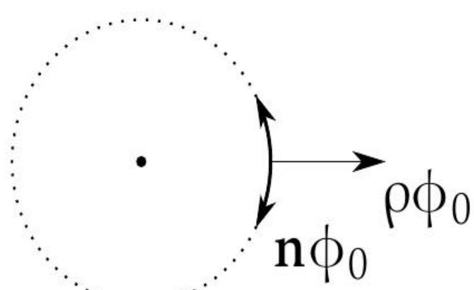
Ch. Ruegg et al., PRL 100,205701 (2008)

Transverse/longitudinal vs amplitude/direction fluctuations (2D)

$\boldsymbol{\varphi} = (\varphi_0 + \sigma)\, \boldsymbol{e}_1 + \boldsymbol{\pi}$



$\boldsymbol{\varphi} = \varphi_0 \sqrt{1+\rho}\, \boldsymbol{n}$

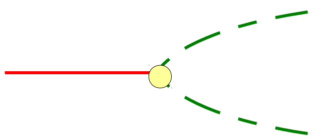


Mean field

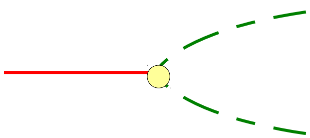
$$G^{\mathrm{MF}}_{\sigma\sigma}(p, i\omega_n) = \frac{1}{\omega_n^2 + c^2 p^2 + \omega_H^2}$$

$$G^{\mathrm{MF}}_{\rho\rho}(p, i\omega_n) = \frac{1}{\omega_n^2 + c^2 p^2 + \omega_H^2}$$

Coupling to Goldstone modes



$\sigma \boldsymbol{\pi}^2$



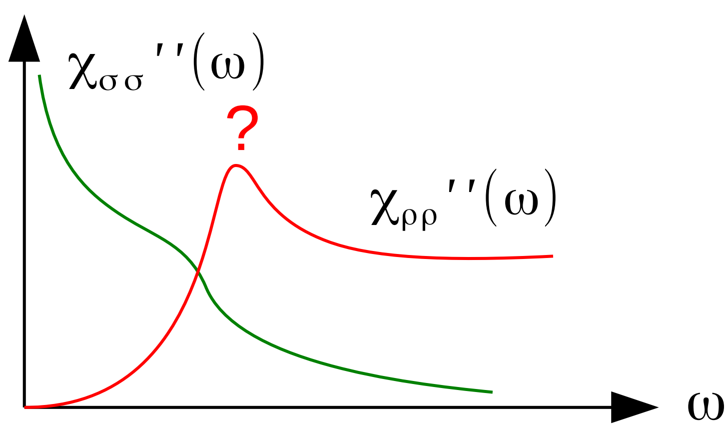
$\rho (\partial_\mu \boldsymbol{n})^2$

Spectral functions (**q**=0):

$$\chi'' = \Im[\chi^R]$$

$$\chi_{\sigma\sigma}''(\omega) \sim \frac{1}{\omega} \quad (\omega \to 0)$$

$$\chi_{\rho\rho}''(\omega) \sim \omega^3 \quad (\omega \to 0)$$



Partial summary

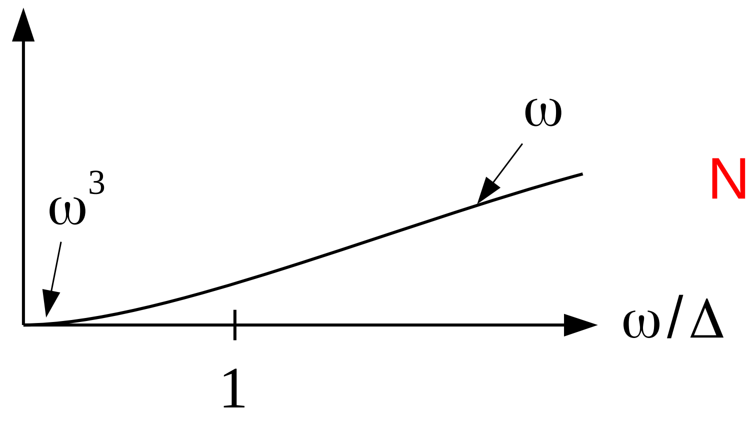
Does the Higgs excitation survive
in the vicinity of the QCP ?

	d=2	d=3
$\chi_{\sigma\sigma}''(\omega)$	no	yes
$\chi_{\rho\rho}''(\omega)$	???	yes

$N \to \infty$:

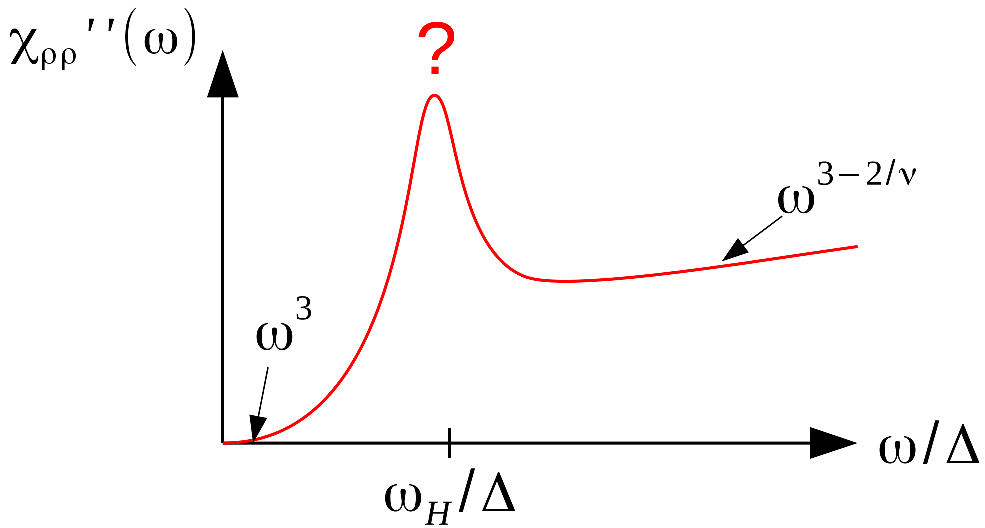
$$\chi_{\rho\rho}''(\omega) \propto \frac{w^3}{w^2 + \left(\frac{4\Delta}{\pi}\right)^2}$$

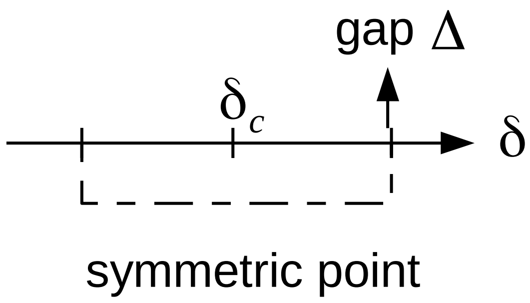
(critical regime)



No resonance!

Universal scaling function (2D)





Δ : characteristic energy scale

$$\chi_{\rho\rho}''(\omega) = A_{\pm}\,\Delta^{3-2/\nu}\,\Phi_{\pm}\left(\frac{\omega}{\Delta}\right)$$

How to compute the spectral function?

$$\chi_{\rho\rho}(\boldsymbol{q}=0, i\omega_n) \rightarrow \chi_{\rho\rho}(\omega + i0^{+}) \text{(analytical continuation)} \rightarrow \chi_{\rho\rho}''(\omega) = \Im\left[\chi_{\rho\rho}(\omega + i0^{+})\right]$$

Non-perturbative (functional) renormalization group

C. Wetterich'93... [reviews: Berges et al.'02, Delamotte'12]

Helmholtz

$$F[h] = -\ln Z[h]$$

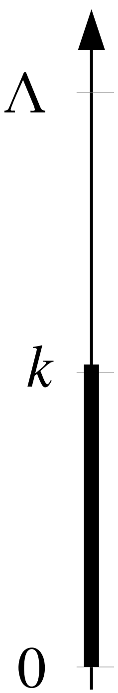
$$Z[h] = \int \mathcal{D}[\varphi] e^{-S[\varphi, g_i] + \int h\varphi}$$

Legendre transform →

Gibbs

$$\phi = \langle \varphi \rangle$$

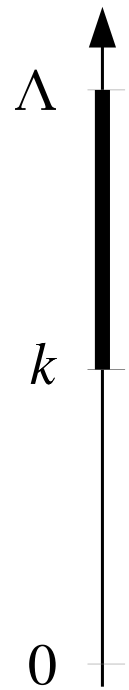
$$\Gamma[\phi] = F[h] - \int dx\, h\, \phi$$



Wilsonian action for the slow modes

$$S[\varphi, g_{i,k}]$$

$$\beta_i(g) = k \frac{dg_{i,k}}{dk}$$



Effective action for the rapid modes

$$\Gamma_k[\phi]$$

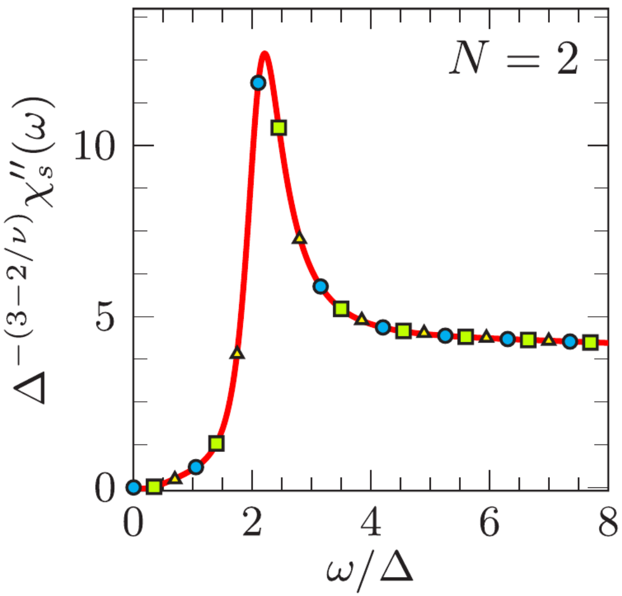
Wilson-Polchinski: $\frac{d}{dk} S[\varphi, g_{i,k}]$

Wetterich's equation: $\frac{d}{dk} \Gamma_k[\phi]$

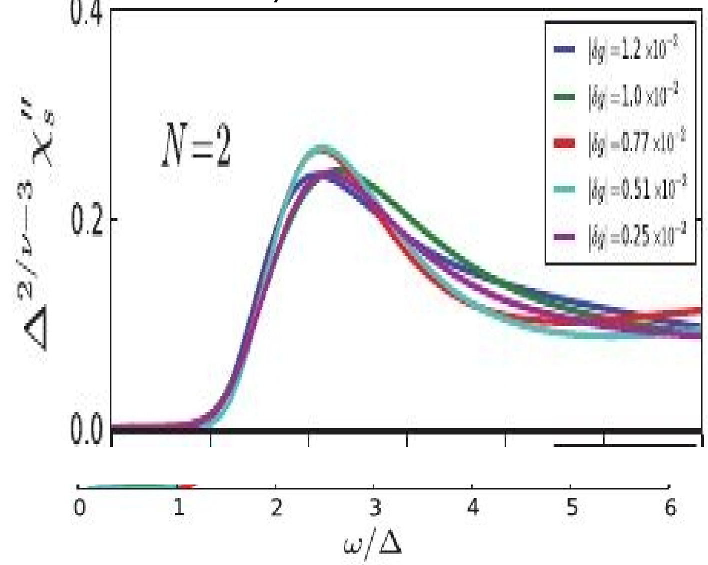
Ordered phase N=2

Non-perturbative RG

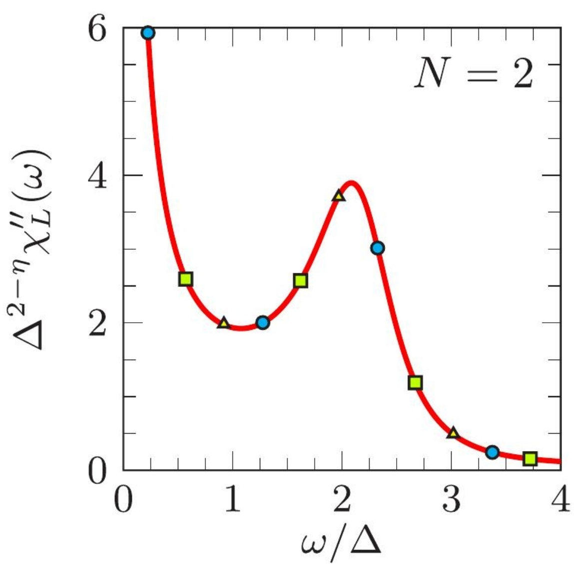
F. Rose, F. Leonnard, ND, PRB'2015



MC: Gazit, Podolsky, Auerbach, Arovas PRL, PRB'2013



Longitudinal spectral function

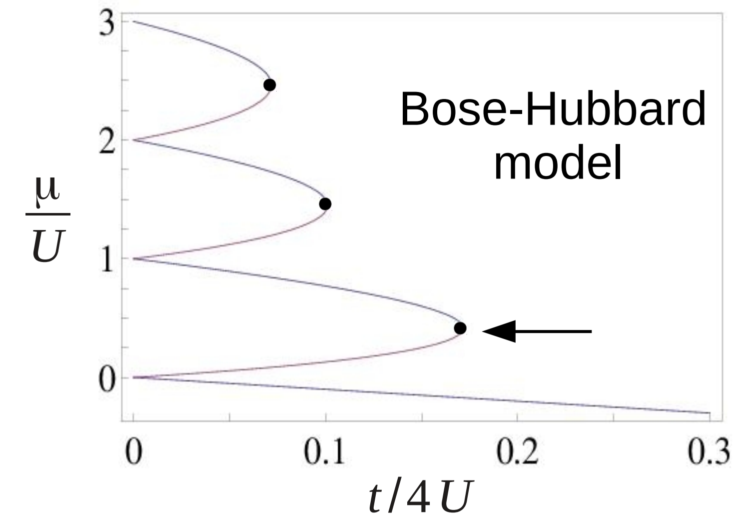


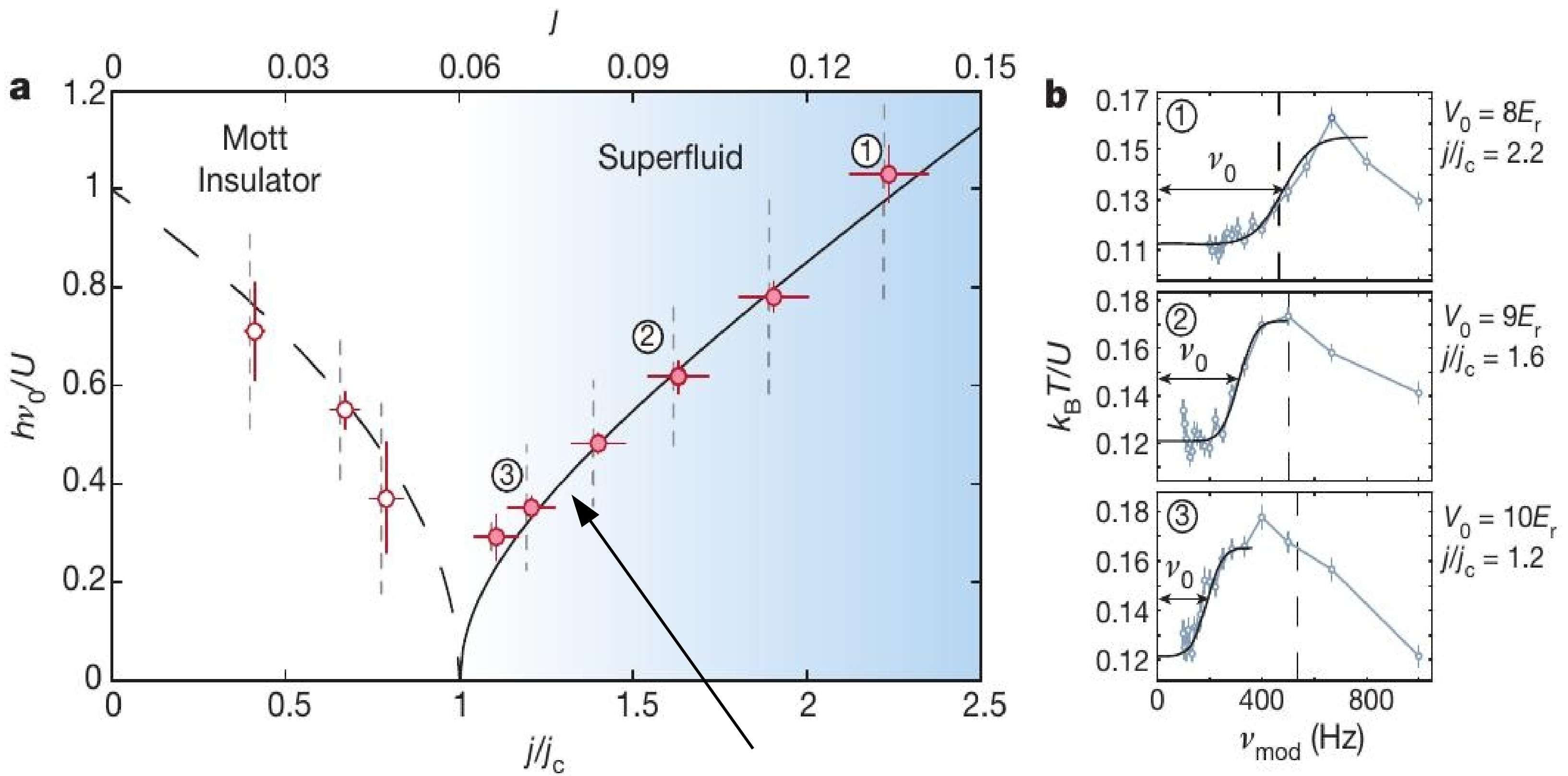
Higgs "mass" ω_H/Δ

N		3	2
NPRG BMW		2.7	2.2
NPRG	[94]		2.4
MC	[93]	2.2(3)	2.1(3)
QMC	[92]		3.3(8)
Perturbative RG	[95]	1.64	1.67
Lattice QMC	[96]	2.6(4)	
Exact diagonalization	[97, 98]	2.7	2.1(2)

Bosons in an optical lattice

M. Endres et al., Nature '2012

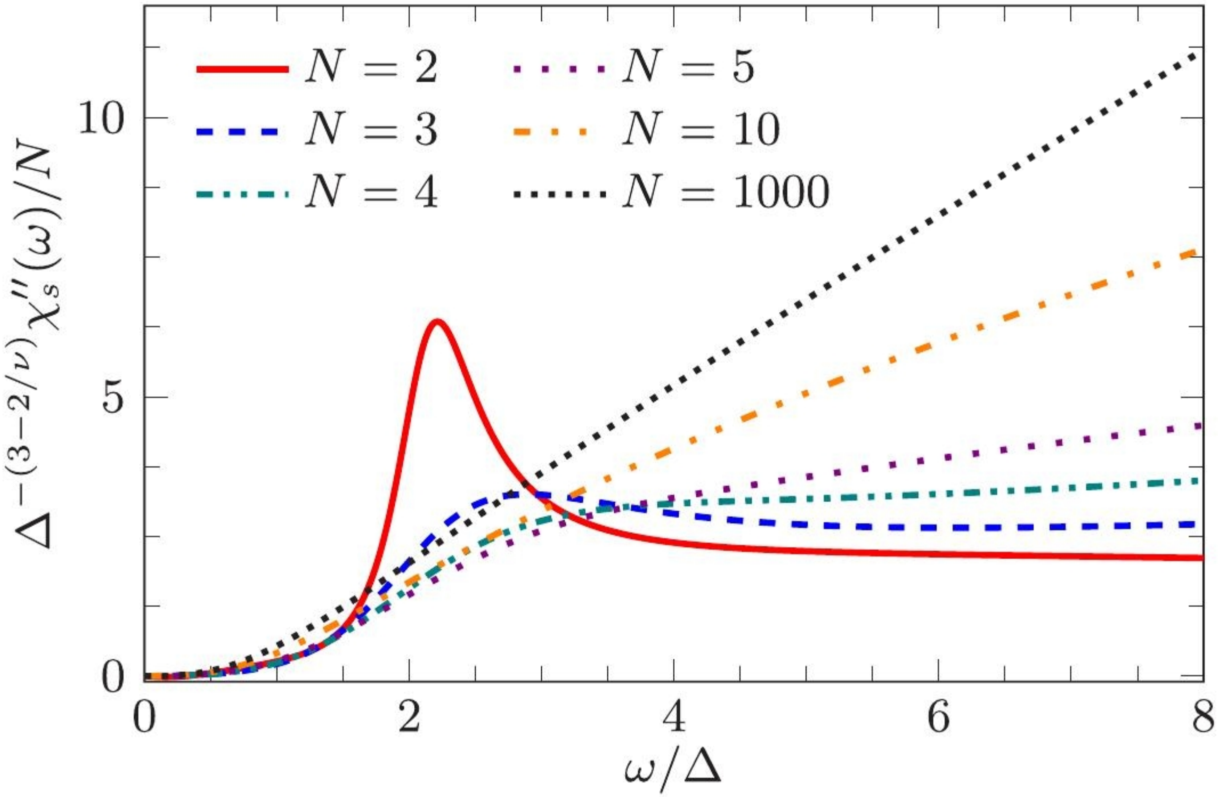




signature of Higgs mode

Temperature response to lattice modulation

Is there a Higgs resonance for larger values of *N* ?



Non-perturbative RG
F. Rose, F. Leonnard, ND
PRB'2015

No Higgs resonance for $N \geq 4$

Conclusion

- Higgs amplitude mode well-defined excitation in 3D (mean-field theory correct).
- The Higgs mode exists for *N*=2 in 2D. Universal features understood from NPRG and QMC ("mass" ω_H and scaling function).
- Related issue: transport near a QCP (current-current correlation function)
  - F. Rose and ND, PRB 95, 014513 (2017)
  - F. Rose and ND, PRB 96, 100501 (2017)
- Experiments: quantum magnets, cold atoms